****

**What does a sinking ship have to do with math?**

**Video:** <https://youtu.be/3m_SgL0JDbE>

**Lesson Plan**

**Teacher Note:** Please preview the entire video and pre-work the solutions in order to anticipate students’ needs, misconceptions and materials unique to your classroom.

You will also need to determine the background knowledge of your students regarding the following science topics, and decide the best method for providing that background in order to support the conceptual understanding of the mathematics shown in the video.

* + Center of Gravity
	+ Center of Buoyancy
	+ Vertical Center of Gravity
	+ Transverse Metacenter

**Common Core Mathematical Content Standards**

* 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
* 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem, understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.
* 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
* Using algebraic methods to solve video question number 4 and use of extension activities would allow the following high school standards to be included:
	+ A.SSE Seeing Structure in Expressions
	+ A.CED Creating Equations
	+ Math Modeling

**Common Core Mathematical Practice Standards**

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively

4. Model with mathematics

6. Attend to precision

**Company Information**

**Fincantieri Marinette Marine (FMM)** was founded in 1942 along the Menominee River in Marinette, Wisconsin to meet America's growing demand for naval construction. From humble beginnings with a contract to build five wooden barges, **FMM** has grown into a world-class shipbuilder, having designed and built more than 1,500 vessels.

Parent company, **FINCANTIERI**, has recently completed a $73.5 million capital expansion program for **Fincantieri Marinette Marine** which has transformed **FMM** into a modern shipbuilding powerhouse, now with 550,000 square feet of manufacturing, warehouse and receiving space, and the capacity to simultaneously build six Littoral Combat Ships in serial production. **FMM** employs cutting-edge computer-controlled manufacturing equipment and has heavy-lift capabilities to meet the most demanding requirement.

**FMM** boasts some of the best engineering and naval architecture minds in the industry, a skilled, safe and motivated workforce, and a management team keenly focused on quality. The company is internationally recognized for innovative and highly efficient, modular, subassembly and assembly-line manufacturing techniques. This sophistication in construction methods has allowed **Fincantieri Marinette Marine** to build some of the most technologically advanced vessels on the planet.

**Fincantieri Marinette Marine’s** performance on government contracts is impressive. Its portfolio includes the U.S. Navy’s Littoral Combat Ship, the improved Navy Lighterage System, mine countermeasure vessels and ocean tugs, as well as U.S. Coast Guard icebreakers, buoy tenders and response vessels. Because of its record of delivering ahead of schedule and within contracted costs, **FMM** has a long-standing relationship with the United States Navy and United States Coast Guard.

**Fincantieri Marinette Marine** is an FOCI mitigated SSA company and is part of the **Fincantieri Marine Group**, the United States division of Italian enterprise **FINCANTIERI**, one of the world's largest shipbuilders with 20 shipyards on four different continents and employing nearly 20,000 shipbuilding professionals. The company has a history dating back 200 years and a track record of producing more than 7,000 ships.

**Summary**

Ship stability is an important area of naval architecture and design. How a ship behaves at sea in a variety of conditions is critical to the safety of crew and cargo. How can we ensure that our ship will float? In this video, you will use a mathematical model for a ship’s center of gravity to determine the stability of the vessel.

**Pre-Activity Discussion:**

In the diagram below:

* Center of gravity of the vessel is G
* Transverse Metacenter is M
* Vertical center of gravity of the vessel is the distance between K and G
* KMt = the distance between K and M
* Extension – (not needed for questions posed in the video) the metacentric height labeled in this diagram determines the level of stability of the ship.

If the ship’s vertical center of gravity in meters is less than its KMt value in meters, the boat is stable and will “right itself” if tilted.



If the ship’s vertical center of gravity in meters is greater than its KMt value in meters, the boat is unstable. If tilted, it would continue in the direction of the tilt.



**Part 1 (0:00 - 2:30)**

**Given Information –**

**Center of Gravity Model (Equation)** - The center of gravity of a ship can be calculated by taking the sum of the moments (forces) and dividing it by the overall weight of the ship. The forces are found by multiplying the weight of each item on the ship and each item’s distance from the bottom of the ship or keel. This distance is called the ***vertical center of gravity***

**Center of Gravity Model**

 OR

zcm = center of gravity of the ship (or center of mass)

N = number of objects on the ship

mi = the mass of each individual object on the ship

zi = the vertical center of gravity of each individual object on the ship.

M = the mass of the entire ship

 **\*\*** **Ship’s KMt value equals 5 meters**

|  |  |  |
| --- | --- | --- |
| Item Description | Mass of Each Item in MT (Metric Tons)Use in formula for the variable :\_\_\_\_\_\_\_\_ | Vertical Center of Gravity of each Item in Meters (m)**Use in formula for the variable:\_\_\_\_\_\_\_\_\_** |
| Hull of Ship | 50 | 2 |
| Cargo | 20 | 5 |
| Engine | 5 | 3 |
| Rudder | 1 | 1 |
| Propeller | 0.5 | 1 |
| Crew | 1 | 10 |
| Lifeboat | 2 | 7 |

* Discuss the model with students. Make connections between the written definition of the model and the algebraic equation.
* Ask students what mathematical operation they think the sigma represents if both models given calculate the same value.
* As the students use the formula, a reminder about labels and unit analysis may be necessary.
* Have students complete the student handout

**Differentiation:**

* The questions on the student handout are scaffolded to meet the needs of students who may need extra support.
* Eliminating some of the added questions and just posing the four questions from the video would be a possible differentiation strategy for students who do not need the extra support.
* Students may also benefit by working with others as part of a partner/group investigation.
* Question 4 may be solved various ways depending on the algebra background of your students.
	+ Substitute strategic values for x until the smallest value is found that makes the inequality true.
	+ Solve the inequality for x

**Part 2: (2:30-3:26)**

* Solutions
* Before showing Part 2, have students share their thinking and solutions with the whole group.
* After showing the solutions, have students reflect on any errors in their thinking and calculations.

**Extension:**

* Investigate summation notation further and explore mathematical problems that are written using summation notation.
	+ Practice using $Σ$ by solving the following problems.



* Investigate what happens to the stability of a ship when **center of gravity = KMt**
	+ **https://www.youtube.com/watch?v=QUgXf2Rj2YQ**

**Student Handout - *What does a sinking ship have to do with math?***  Name(s):

**Pre-Video Discussion:**  *What determines a ship’s stability?*

**Problem:** *Will the ship described in the video be stable as it transports its cargo across Lake Michigan?*

**Given information:**

**Center of Gravity Model (Equation)** - The center of gravity of a ship can be calculated by taking the sum of the moments (forces) and dividing it by the overall weight of the ship. The forces are found by multiplying the weight of each item on the ship and each item’s distance from a reference point. This distance is called the ***vertical center of gravity*** and the reference point used is the bottom of the ship or keel.

**Center of Gravity Model**

 OR

zcm = center of gravity of the ship (or center of mass)

n = number of objects on the ship

mi = the mass of each individual object on the ship

zi = the vertical center of gravity of each individual object on the ship.

M = the mass of the entire ship

 **\*\*Ship’s KMt value equals 5 meters**

|  |  |  |
| --- | --- | --- |
| Item Description | Mass of Each Item in MT (Metric Tons)**Use in formula for the variable :\_\_\_\_\_\_\_** | Vertical Center of Gravity of each Item in Meters (m)**Use in formula for the variable:\_\_\_\_\_\_\_\_\_** |
| Hull of Ship | 50 | 2 |
| Cargo | 20 | 5 |
| Engine | 5 | 3 |
| Rudder | 1 | 1 |
| Propeller | 0.5 | 1 |
| Crew | 1 | 10 |
| Lifeboat | 2 | 7 |

1. Fill in the table headings with the correct variable from the model.
2. What will this model help you calculate?
3. What is the name of the math symbol $Σ$ and what does it tell you to do?
4. How can you find M, the mass of the entire ship? (This is question number 1 in the video)
5. Using the order of operations, which calculation in the model should you do first?
6. Use the model to find the vertical center of gravity of the whole ship. (This is question number 2 in the video)
7. Using the background information, you learned about ship stability, the vertical center of gravity of the whole ship and the KMt value given, will this ship be stable? Justify your reasoning. (This is question number 3 in the video)
8. The weight of the crew may change depending on number of crew members and their individual weights. How heavy would the crew have to be for the ship to be unstable? (This is question number 4 in the video)

**ANSWER KEY:**

1. mi , zi

1. The vertical center of gravity of the whole ship (Zcm)

1. The Greek letter sigma is a symbol for summation. We use it to add of a sequence of numbers.

1. Find the sum of the masses of all the items listed in the table. 79.5 MT

1. Multiply the mass and vertical center of gravity for each item in the table.

1. Zcm = $\frac{(50\*2) + (20\*5) + (5\*3) + (1\*1) + (0.5\* 1) + (1\*10) + (2\*7)}{79.5}$

Zcm = $\frac{240.5 MTm}{79.5 MT}$

Zcm  = 3.03 meters

G. The ship’s vertical center of gravity is 3.03 meters. This is the distance from the keel to the center of gravity. The ship’s KMt is 5 meters. This is the distance from the keel to the transverse metacenter.

 3.03 meters < 5 meters Yes, the ship is stable.

 H. Let x = the mass of the crew in metric tons

 The formula from above is now:

 $\frac{100+100+15+1+0.5+10x+14}{78.5 + x}$ > 5m

 $\frac{230,5 + 10x}{78.5 + x}$ > 5m

x > 32.4 MT